08

Mechanical Properties of Solids

TOPIC 1

Stress, Strain and Hooke's Laws

01 A wire of length L, area of cross-section A is hanging from a fixed support. The length of the wire changes to L_1 when mass M is suspended from its free end. The expression for Young's modulus is **[NEET (Sep.) 2020]**

(a) $\frac{Mg(L_1 - L)}{AL}$ (b) $\frac{MgL}{AL_1}$

(c) $\frac{MgL}{A(L_1 - L)}$ (d) $\frac{MgL_1}{AL}$

Ans. *(c)*

Here, change in length, $\Delta L = (L_1 - L)$

Area $= A$

Force, $F = Mg$

Young's modulus, $Y = \frac{\text{Normal stress}}{\text{Longitudinal strain}}$

$$\Rightarrow \quad Y = \frac{(F/A)}{\left(\frac{\Delta L}{L}\right)} = \frac{\frac{Mg}{A}}{\left(\frac{L_1 - L}{L}\right)} = \frac{MgL}{A(L_1 - L)}$$

Hence, correct option is (c).

02 Two wires are made of the same material and have the same volume. The first wire has cross-sectional area A and the second wire has cross-sectional area $3A$. If the length of the first wire is increased by Δl on applying a force F, how much force is needed to stretch the second wire by the same amount? **[NEET 2018]**

(a) $4F$ (b) $6F$ (c) $9F$ (d) F

Ans. *(c)*

According to the question,

For wire 1

Area of cross-section $= A_1$

Force applied $= F_1$

Increase in length $= \Delta l$

From the relation of Young's modulus of elasticity,

$$Y = \frac{Fl}{A\Delta l}$$

Substituting the values for wire 1 in the above relation, we get

$$\Rightarrow \quad Y_1 = \frac{F_1 l_1}{A_1 \Delta l} \quad \text{...(i)}$$

For wire 2

Area of cross-section $= A_2$

Force applied $= F_2$

Increase in length $= \Delta l$

Similarly,

$$Y_2 = \frac{F_2 l_2}{A_2 \Delta l} \quad \text{...(ii)}$$

$\because$ Volume, $V = Al$

or $l = \frac{V}{A}$

Substituting the value of l in Eqs. (i) and (ii), we get

$$Y_1 = \frac{F_1 V}{A_1^2 \Delta l} \text{ and } Y_2 = \frac{F_2 V}{A_2^2 \Delta l}$$

As it is given that the wires are made up of same material, i.e. $Y_1 = Y_2$

$$\Rightarrow \quad \frac{F_1 V}{A_1^2 \Delta l} = \frac{F_2 V}{A_2^2 \Delta l}$$

$$\Rightarrow \quad \frac{F_1}{F_2} = \frac{A_1^2}{A_2^2} = \frac{A^2}{9A^2}$$

$(\because A_1 = A \text{ and } A_2 = 3A)$

$$= \frac{1}{9}$$

or $F_2 = 9F_1 = 9F$ (given, $F_1 = F$)

03 The bulk modulus of a spherical object is B. If it is subjected to uniform pressure p, the fractional decrease in radius is **[NEET 2017]**

(a) $\frac{p}{B}$ (b) $\frac{B}{3p}$

(c) $\frac{3p}{B}$ (d) $\frac{p}{3B}$

Ans. *(d)*

The object is spherical and the bulk modulus is represented by B. It is the ratio of normal stress to the volumetric strain.

Hence $B = \frac{F/A}{\Delta V/V} \Rightarrow \frac{\Delta V}{V} = \frac{p}{B}$

$$\Rightarrow \quad \left|\frac{\Delta V}{V}\right| = \frac{p}{B}$$

Here p is applied pressure on the object and $\frac{\Delta V}{V}$ is volume strain

Fractional decreases in volume

$$\Rightarrow \quad \frac{\Delta V}{V} = 3\frac{\Delta R}{R} \quad [\because V = \frac{4}{3}\pi R^3]$$

Volume of the sphere decreases due to the decrease in its radius.

Hence $\frac{\Delta V}{V} = \frac{3\Delta R}{R} = \frac{p}{B}$

$$\Rightarrow \quad \frac{\Delta R}{R} = \frac{p}{3B}$$

04 Copper of fixed volume V is drawn into wire of length l. When this wire is subjected to a constant force F, the extension produced in the wire is Δl. Which of the following graphs is a straight line? **[CBSE AIPMT 2014]**

(a) Δl versus $\frac{1}{l}$ (b) Δl versus l^2

(c) Δl versus $\frac{1}{l^2}$ (d) Δl versus l

Ans. *(b)*

Youngs' modulus is given by

$$Y = \frac{F \times l}{A \times \Delta l} \quad \text{...(i)}$$

As $V = A \times l = \text{constant}$ ...(ii)

From Eqs. (i) and (ii), we get

$$Y = \frac{F \times l^2}{V \times \Delta l} \Rightarrow \Delta l = \frac{F}{V \times Y} \times l^2$$

$\Rightarrow \Delta l \propto l^2$

05 The following four wires are made of same material. Which of these will have the largest extension when the same tension is applied? **[NEET 2013]**

(a) Length = 50 cm, diameter = 0.5 mm
(b) Length = 100 cm, diameter = 1 mm
(c) Length = 200 cm, diameter = 2 mm
(d) Length = 300 cm, diameter = 3 mm

Ans. *(a)*

As $\gamma = \frac{F \times L}{\Delta L \times A} = \frac{mg \cdot L}{\Delta L \cdot A}$ or $\Delta L = \frac{mgL}{\gamma \cdot A}$

$\Rightarrow \Delta L \propto \frac{L}{A}$, which is maximum for option (a).

TOPIC 2

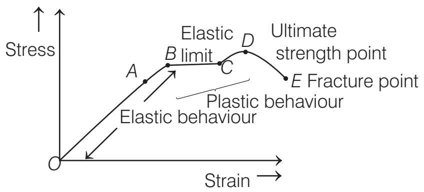
Stress-Strain Curve, Thermal Stress and Elastic PE

06 The stress-strain curves are drawn for two different materials X and Y. It is observed that the ultimate strength point and the fracture point are close to each other for material X but are far apart for material Y. We can say that materials X and Y are likely to be (respectively) **[NEET (Odisha) 2019]**

(a) ductile and brittle
(b) brittle and ductile
(c) brittle and plastic
(d) plastic and ductile

Ans. *(b)*

The stress-strain curve for a material is shown



This curve specifies the behaviour of material.

For the material, if distance between strength point and fracture point is small, so it is brittle and will break easily on the application of some extra stress after point D.

For material Y, if the distance between strength point and fracture point is large, so it is a ductile material and can withstand for some extra stress beyond point D.

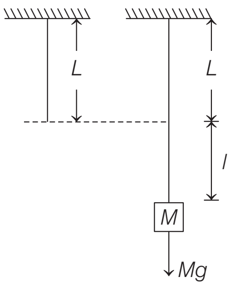
07 When a block of mass M is suspended by a long wire of length L, the length of the wire becomes $(L + l)$. The elastic potential energy stored in the extended wire is **[NEET (National) 2019]**

(a) MgL (b) $\frac{1}{2}Mgl$

(c) $\frac{1}{2}MgL$ (d) Mgl

Ans. *(b)*

In stretching a wire, the work done against internal restoring force is stored as elastic potential energy in wire and given by



$$U = W = \frac{1}{2} \times \text{Force}(F) \times \text{Elongation}(l)$$

$$= \frac{1}{2}Fl = \frac{1}{2} \times Mg \times l = \frac{1}{2}Mgl$$